HORNSBY GIRLS HIGH SCHOOL

Mathematics Extension 1

Year 12 Higher School Certificate

Trial Examination Term 3 2023

STUDENT NUMBER: ____________________

STUDENT NAME:____________________ **TEACHER NAME:** ____________________

General Instructions:

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- In Questions 11–14, show relevant mathematical reasoning and/ or calculations

Total Marks: 70

Section I – 10 marks (pages 3–6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 7–11)

- Attempt Questions 11–14
- Start each question in a new writing booklet
- Write your student number on every writing booklet
- Allow about 1 hour and 45 minutes for this section

Question	*1-10*	*11*	*12*	*13*	*14*	*Total*
Total	*/10*	*/17*	*/15*	*/14*	*/14*	*/70*

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

Question 1

What is the remainder when $P(x) = x^3 + 5x^2 + 5x - 2$ is divided by $(x-2)$?

(A) -2

(B) 0

(C) 18

(D) 36

Question 2

Which of the following in the inverse function of $f(x) = \dfrac{1}{x+2}, \quad x > -2$?

(A) $y = \dfrac{1}{x} - 2, \quad x > -2$

(B) $y = \dfrac{1}{x} - 2, \quad x > 0$

(C) $y = \dfrac{1}{x-2}, \quad x < -2$

(D) $y = \dfrac{1}{x-2}, \quad x < 0$

Question 3

Which of the following is the solution of $\dfrac{x}{3-x} \leq 2$?

(A) $x \leq 2$

(B) $x < 2, x > 3$

(C) $x \leq 2, x > 3$

(D) $x \leq 2, x \geq 3$

Question 4

How many arrangements of the letters of the word PROBABILITY are possible if the B's are together and the arrangement does not start or end with a vowel?

(A) $\dfrac{7\times6\times9!}{2!\times2!}=3\,810\,240$

(B) $6\times5\times8!=1\,209\,600$

(C) $\dfrac{6\times5\times8!}{2!}=604\,800$

(D) $\dfrac{6\times5\times8!}{2!\times2!}=302\,400$

Question 5

Which of the following is the exact value of $\displaystyle\int_1^2 \frac{3}{\sqrt{4-x^2}}\,dx$?

(A) $\dfrac{\pi}{6}$

(B) $\dfrac{\pi}{3}$

(C) $\dfrac{\pi}{2}$

(D) π

Question 6

A curve is represented by the parametric equations $x=\ln t$ and $y=3t^2-5t$.

Which of the following is an expression for $\dfrac{dy}{dx}$ in terms of t?

(A) $\dfrac{6t-5}{t}$

(B) $\dfrac{t}{6t-5}$

(C) $t(6t-5)$

(D) $\dfrac{1}{t(6t-5)}$

Question 7

Which of the following is an expression for $\int \sin^2 4x\,dx$?

(A) $\dfrac{x}{2}+\dfrac{\sin 8x}{16}+c$

(B) $\dfrac{x}{2}-\dfrac{\sin 8x}{16}+c$

(C) $\dfrac{x}{2}+\dfrac{\cos 8x}{16}+c$

(D) $\dfrac{x}{2}-\dfrac{\cos 8x}{16}+c$

Question 8

Which of the following is equivalent to $\dfrac{d}{dx}\left(\tan^{-1}\dfrac{3x}{2}\right)$?

(A) $\dfrac{3}{4+3x^2}$

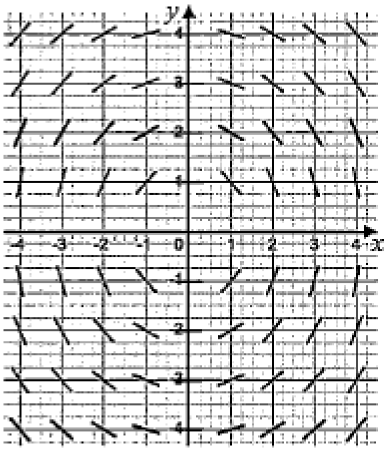
(B) $\dfrac{6}{4+3x^2}$

(C) $\dfrac{3}{4+9x^2}$

(D) $\dfrac{6}{4+9x^2}$

Question 9

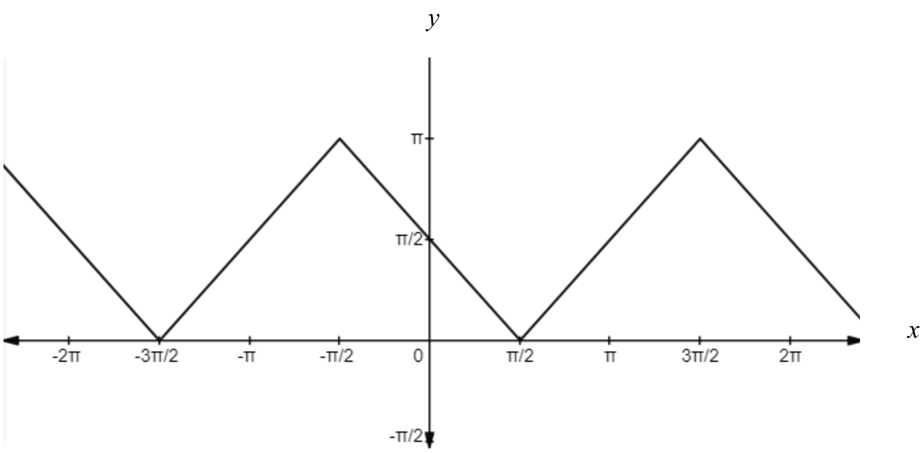
Which differential equation is represented by the following slope field?



(A) $\frac{dy}{dx}=\frac{-x}{y}$ (B) $\frac{dy}{dx}=\frac{-y}{x}$ (C) $\frac{dy}{dx}=\frac{x}{y}$ (D) $\frac{dy}{dx}=\frac{y}{x}$

Question 10

Which of the following equations is shown in the sketch below?



(A) $y=\sin^{-1}(\cos x)$

(B) $y=\sin(\cos x^{-1})$

(C) $y=\cos(\sin x^{-1})$

(D) $y=\cos^{-1}(\sin x)$

Section II

60 marks
Attempt Questions 11 to 14
Allow about 1 hour and 45 minutes for this section
Instructions

- Answer the questions in the appropriate writing booklet.
- In Questions 11-14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (17 marks) Start a new writing booklet.

(a) A committee containing 4 women and 4 men is to be formed from a group of 6 women and 7 men.

(i) In how many different ways can the committee be formed? **1**

(ii) After the committee has been formed, the 4 women and 4 men are seated around a circular table. If the seats are randomly assigned, find the probability that a particular man and woman are not seated together and men and women alternate around the table. **2**

(b) Two of the roots of the equation $3x^3 - ax^2 - 17x + 6 = 0$ are reciprocals of one another. Find the value of the three roots and the value of a. **3**

(c) Using $t = \tan\frac{\theta}{2}$, solve $2\cos\theta + 3\sin\theta = -2$ for $0 \le \theta \le 2\pi$. **3**

(d) In the expansion of $(1+2x)^n$, the coefficients of x^6 and x^8 are in the ratio 7:3. Find the value of n. **3**

(e) (i) For positive integers n and r, with $r < n$, show that ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$. **2**

(ii) Hence, prove, by Mathematical Induction, that $\sum_{j=3}^{n} {}^{j-1}C_2 = {}^nC_3$ for all integers $n \ge 3$. **3**

End of Question 11

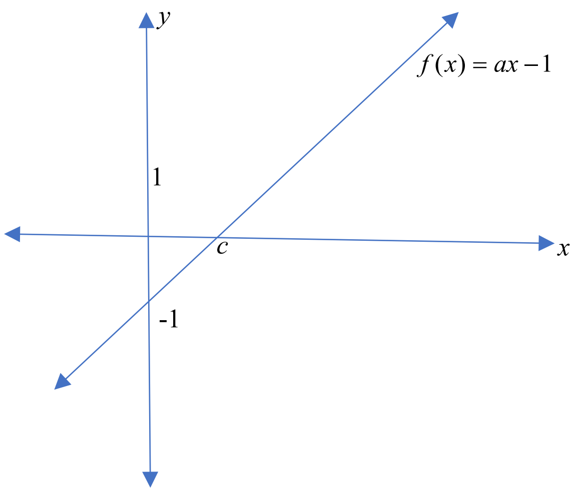
Question 12 (15 marks) Start a new writing booklet.

(a) A spherical object is heated so that its volume is expanding at the rate of 40π mm^3/s.

(i) Find the rate of change of the radius when $r = 3$ mm. **1**

(ii) Hence, find the rate of change of the surface area when $r = 3$ mm. **1**

(b) The diagram shows the graph of $f(x) = ax - 1$



(i) Copy the diagram into your writing booklet and sketch the graph of $y = \dfrac{1}{f(x)}$ on the same number plane. **2**

(ii) The solution of the inequality $f(x) \geq \dfrac{1}{f(x)}$ is $x \in [0, c) \cup [3, \infty)$. **1**

Find the value of a.

(c) A sports association manages 12 junior teams. It decides to check the age of all players. **2**
Any team that has more than three players above the age limit will be penalised.

A total of 38 players are found to be above the age limit.
Will any team be penalised? Justify your answer.

(d) (i) Express $2\sin x+\sqrt{5}\cos x$ in the form $R\sin(x+\alpha)$ for $R>0$ and $0^0 \le \alpha \le 90^0$. **2**

(ii) Hence, solve the equation $2\sin x+\sqrt{5}\cos x=1$ for $0^0 \le \theta \le 360^0$. **2**

(e) The position of a boat, relative to a stationary observer at the origin, point *O*, is being recorded.

The boat travels at a constant speed and in a constant direction and passes through three collinear points *A*, *B* and *C*. The boat is first observed at point *A* on a position vector of $-20\underset{\sim}{i}-12\underset{\sim}{j}$.

One minute later, the boat is observed at point *B* on a position vector of $-5\underset{\sim}{i}+5\underset{\sim}{j}$.

Using binoculars and without taking their eyes off the boat, the observer watches the motion of the boat for five minutes, starting when the boat is at point *A*, passing through the point *B* and continuing through to the point *C*.

(i) Find the position vector of the boat after the five minutes when it is at point *C*. **2**

(ii) Hence, find $\angle AOC$, the angle through which the observer's binoculars move during this five minute interval, correct to the nearest degree. **2**

End of Question 12

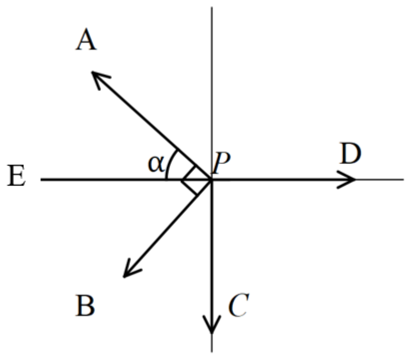
Question 13 (14 marks) Start a new writing booklet.

(a) Find $\int_{\frac{\pi}{2}}^{\pi} \cos 5x \sin 3x \; dx$. **3**

(b) Find the equation of the curve which satisfies the differential equation $\frac{dy}{dx} = \frac{1-x}{y}$ and passes through the point (2,0). **3**

(c) An object is in equilibrium under the action of four forces $\overrightarrow{PA}$, $\overrightarrow{PB}$, $\overrightarrow{PC}$ and $\overrightarrow{PD}$ acting in the direction shown in the diagram below. $\left|\overrightarrow{PA}\right| = 7$ N, $\left|\overrightarrow{PB}\right| = 5$ N, $\left|\overrightarrow{PC}\right| = \left|\overrightarrow{PD}\right| = F$ N.

$\angle APB = 90°$, $\angle APE = \alpha$.



(i) Show that $F = 7\cos\alpha + 5\sin\alpha$ and $F = 7\sin\alpha - 5\cos\alpha$. **4**

(ii) Hence, determine the value of α, correct to the nearest degree. **1**

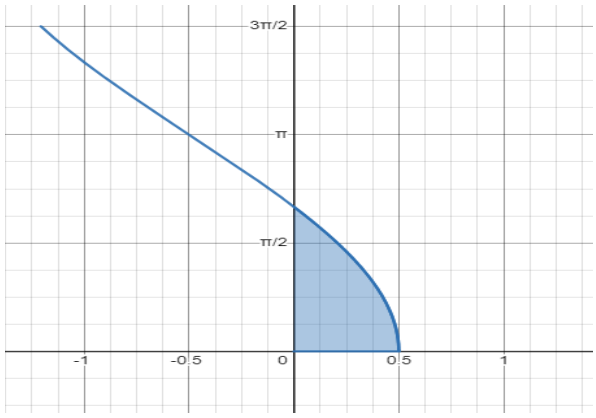
(d) The area bounded by the curve $y = e^x \sqrt[4]{1+e^{2x}}$ is rotated about the x-axis from $x = -1$ to $x = 1$. Find the volume generated, expressing your answer correct to 2 significant figures. **3**

End of Question 13

Question 14 (14 marks) Start a new writing booklet.

(a) Consider the function $f(x) = 2\sin^{-1}\left(\dfrac{x}{3}\right) + \dfrac{\pi}{2}$. State the domain and range of $f(x)$. **2**

(b) The graph of $y = 2\cos^{-1}(x + \frac{1}{2})$ is shown below.

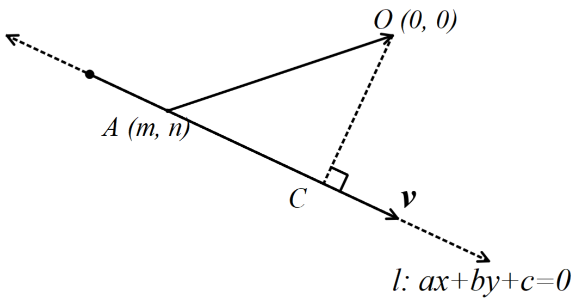


Calculate the shaded area. **3**

(c) Water is flowing into an empty tank. After ***t*** hours, the rate of change of the volume of water with respect to time is given by the differential equation $\dfrac{dV}{dt} = \dfrac{3(2 - V^2)}{V}$. **4**

Find the volume of water in the tank after one hour.

(d) Point $A\ (m,\ n)$ is on the line $l:\ ax + by + c = 0$. The values of *m, n, a, b, c* are all real constants. Vector $\underset{\sim}{v}$ overlaps line *l*.



(i) Show that $\underset{\sim}{v} = \begin{pmatrix} -b \\ a \end{pmatrix}$ is a vector that overlaps line *l*. **1**

(ii) Using vector projection, show that the perpendicular distance from *O* to line *l* is $\dfrac{|c|}{\sqrt{a^2 + b^2}}$. **4**

End of paper

Hornsby Girls High School
Year 12 Mathematics Extension 1 HSC Trial 2023
Solutions

Multiple Choice

Solutions	Marker's Comments
Question 1 D $P(2) = 2^3 + 5(2)^2 + 5(2) - 2$ $= 36$	
Question 2 B $y = \frac{1}{x+2}, \quad x > -2, y > 0$ $x = \frac{1}{y+2}, \quad y > -2, x > 0$ $y + 2 = \frac{1}{x}$ $y = \frac{1}{x} - 2, y > -2, x > 0$	
Question 3 C $\frac{x}{3-x} \times (3-x)^2 \leq 2(3-x)^2$ $x(3-x) \leq 2(3-x)^2$ $x(3-x) - 2(3-x)^2 \leq 0$ $(x - 2(3-x))(3-x) \leq 0$ $(x - 6 + 2x)(3-x) \leq 0$ $(3x-6)(3-x) \leq 0$ $3(x-2)(x-3) \geq 0$ $x \leq 2, x > 3 \quad (\text{note: } x \neq 3)$	
Question 4 C 11 letters, 2Bs, 2Is, 4 vowels, 7 consonants For the Bs to be together, treat them as a single element along with the other 9 letters. First consonant (B or others) Middle last consonant $6 \times \frac{8!}{2!} \times 5$	

Solutions	Marker's Comments
Question 5 D $\int_1^2 \frac{3}{\sqrt{4-x^2}}\,dx$ $=3\left[\sin^{-1}\frac{x}{2}\right]_1^2$ $=3\left(\sin^{-1}\frac{2}{2}-\sin^{-1}\frac{1}{2}\right)$ $=3\left(\frac{\pi}{2}-\frac{\pi}{6}\right)$ $=\pi$	
Question 6 C $x=\ln t, y=3t^2-5t$ $\frac{dx}{dt}=\frac{1}{t}, \frac{dy}{dt}=6t-5$ $\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}$ $=t(6t-5)$	
Question 7 B $\int \sin^2 4x\,dx=\frac{1}{2}\int(1-\cos 8x)\,dx$ $=\frac{1}{2}\left(x-\frac{\cos 8x}{8}\right)+C$ $=\frac{x}{2}-\frac{\cos 8x}{16}+C$	
Question 8 D $\frac{d}{dx}\left(\tan^{-1}\frac{3x}{2}\right)=2\times\frac{3}{2^2+(3x)^2}$ $=\frac{6}{4+9x^2}$ *or* $=\frac{\frac{3}{2}}{1+\left(\frac{3x}{2}\right)^2}$ $=\frac{6}{4+9x^2}$	

Solutions	Marker's Comments
Question 9 A Inspect the slope field: $x=0, \frac{dy}{dx}=0,$ eliminate B and D if x and y have same signs (both + or both -), $\frac{dy}{dx}<0$ if x and y have different signs, $\frac{dy}{dx}>0$	
Question 10 D The domain on the graph is all real x. Not B or C. For A, $x=\frac{\pi}{2}, \cos\frac{\pi}{2}=0, \sin^{-1}0=\frac{\pi}{2}$. NOT A. For A, For D, $x=\frac{\pi}{2}, \sin\frac{\pi}{2}=1, \cos^{-1}1=0.$ $x=0, \sin 0=0, \cos^{-1}0=\frac{\pi}{2}.$	

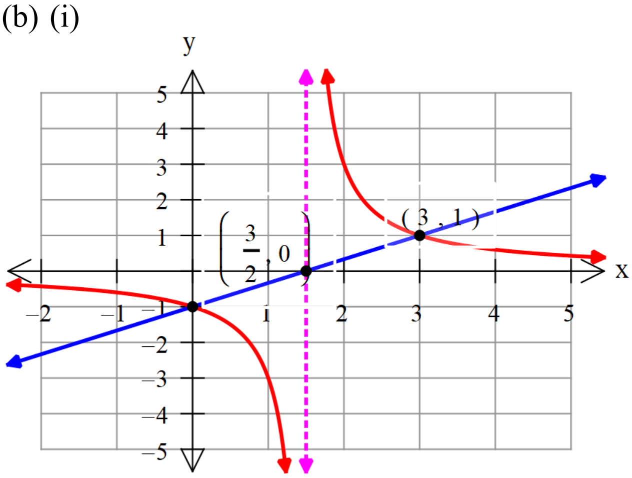
SECTION II

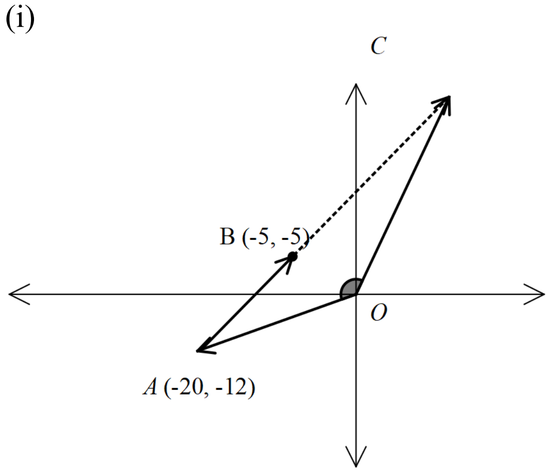
Solutions	Marker's Comments
Question 11 (a) (i) $^6C_4\times{}^7C_3=15\times35$ $=525$	Mostly well done. A few added the combinations rather than multiplying.
(a) (ii) $\frac{1\times3!\times2\times3!}{7!}=\frac{72}{5040}$ $=\frac{1}{70}$	This question was poorly done. A few students only worked out how many and not the probability Aw 1 for correct numerator or denominator Aw 2 for correct fraction

(b) $3x^3 - ax^2 - 17x + 6 = 0$ have roots $\alpha,\ \frac{1}{\alpha},\ \beta$ $\alpha + \frac{1}{\alpha} + \beta = \frac{a}{3}$ (1) $\alpha \times \frac{1}{\alpha} + \alpha\beta + \beta \times \frac{1}{\alpha} = \frac{-17}{3}$ (2) $\alpha \times \frac{1}{\alpha} \times \beta = \frac{-6}{3}$ $\beta = -2$ sub $\beta = -2$ into (2): $1 + \alpha(-2) + (-2) \times \frac{1}{\alpha} = \frac{-17}{3}$ $6\alpha^2 - 20\alpha + 6 = 0$ $3\alpha^2 - 10\alpha + 2 = 0$ $(3\alpha - 1)(\alpha - 3) = 0$ $\alpha = 3,\ \frac{1}{3}$ sub $\alpha = 3$ (or $\frac{1}{3}$), $\beta = -2$ into (1): $\frac{1}{3} + 3 - 2 = \frac{a}{3}$ $a = 4$	Mostly well done except for those who had algebraic errors. Aw 1 for Beta=-2 or equivalent Aw 2 for Beta=-2 and either alpha=3 or a=4 Aw 3 for all 4 answers correct
Question 11 (c) $2\frac{1-t^2}{1+t^2} + 3\frac{2t}{1+t^2} = -2$ $2(1-t^2) + 6t + 2(1+t^2) = 0$ $2 - 2t^2 + 6t + 2 + 2t^2 = 0$ $t = \frac{-2}{3}$ $\tan\frac{\theta}{2} = \frac{-2}{3}, 0 \le \theta \le 2\pi$ $\frac{\theta}{2} = \pi - \tan^{-1}\left(\frac{2}{3}\right),\ 0 \le \frac{\theta}{2} \le \pi$ $\theta = 2\left(\pi - \tan^{-1}\left(\frac{2}{3}\right)\right)$ ≈ 5.1 Check $\theta = \pi$, left hand side $= 2\cos\pi + 2\sin\pi$ $= 2(-1) + 2(0)$ $= -2$ $=$ *right* hand side $\therefore$ The solutions are: $\theta = \pi, \theta \approx 5.1$	Aw 1 for t=-2/3 Aw 2 for either theta=5.1 or checking pi as a solution Some students did not restrict theta/2 to 1st and 2nd quads. Some expressed answers in degrees when the domain is in radians Aw 3 for both correct answers

(d) $$(1+2x)^n=\sum_{r=0}^{n}{}^nC_r1^{n-r}(2x)^r$$ $$\frac{{}^nC_6(2x)^6}{{}^nC_8(2x)^8}=\frac{7}{3}$$ $$\frac{\dfrac{n!}{6!(n-6)!}\times 2^6}{\dfrac{n!}{8!(n-8)!}\times 2^8}=\frac{7}{3}$$ $$\frac{8!(n-8)!}{6!(n-6)!2^2}=\frac{7}{3}$$ $$\frac{8\times 7}{4(n-6)(n-7)}=\frac{7}{3}$$ $$6=(n-6)(n-7)$$ $$n^2-13n+36=0$$ $$(n-9)(n-4)=0$$ $$\therefore n=9 \quad (n\geq 8)$$	Aw 1 for this step A lot of students thought (n-8)!>(n-6)! This is not the case. (n-6)!>(n-8)! Some students guessed an answer of n=9 (This was Aw 2 only as the other answer of n=4 was not discussed and omitted) Aw 3 for correct solution
Question 11 (e) (i) $$LHS={}^nC_r+{}^nC_{r+1}$$ $$=\frac{n!}{r!(n-r)!}+\frac{n!}{(r+1)!(n-r-1)!}$$ $$=n!\frac{(r+1)+(n-r)}{(r+1)!(n-r)!}$$ $$=\frac{n!(n+1)}{(r+1)!(n-r)!}$$ $$=\frac{(n+1)!}{(r+1)!(n-r)!}$$ $$=\frac{(n+1)!}{(r+1)!(n+1-(r+1))!}$$ $$={}^{n+1}C_{r+1}$$ $$=RHS$$	This was generally well done. However, there were a few students who do not know how to get the first mark of finding the LCD of factorials.

(e) (ii) $\sum_{j=3}^{n} {}^{j-1}C_2 = {}^{n}C_3$ ${}^{3-1}C_2 + {}^{4-1}C_2 + {}^{5-1}C_2 + ... + {}^{n-1}C_2 = {}^{n}C_3$ ${}^{2}C_2 + {}^{3}C_2 + {}^{4}C_2 + ... + {}^{n-1}C_2 = {}^{n}C_3$ For $n = 3$, LHS $= {}^{3-1}C_2$ $= {}^{2}C_2$ $= 1$ $RHS = {}^{3}C_3$ $= 1$ $\therefore$ Proven true for $n = 3$ Assume true for $n = k$, ${}^{2}C_2 + {}^{3}C_2 + {}^{4}C_2 + ... + {}^{k-1}C_2 = {}^{k}C_3$ Required to prove true for $n = k+1$, $RTP: {}^{2}C_2 + {}^{3}C_2 + {}^{4}C_2 + ... + {}^{k-1}C_2 + {}^{k}C_2 = {}^{k+1}C_3$ $LHS = {}^{2}C_2 + {}^{3}C_2 + {}^{4}C_2 + ... + {}^{k-1}C_2 + {}^{k}C_2$ $= {}^{k}C_3 + {}^{k}C_2$ (as shown in part ii) $= {}^{k+1}C_3$ $= RHS$ If true for $n = k$, proven true for $n = k+1$. Since true for $n = 3$, true for $n = 3+1 = 4,\ n = 4+1 = 5,\ ......$ Therefore true for positive integers, $n \geq 3$.	Those who looked at the pattern of the sum generally did better than those who didn't. A few of Those who continued to use the sum notation struggled to understand and even a few left it off in step 2 and 3 which made no sense. Aw 1 for one correct step Aw 2 for 2 correct steps Aw 3 for All correct steps Bringing in (i) made this step very easy. Some used the factorial notation but this made it harder and quiet often led to errors. Some need to learn how to write their conclusions better. While marks are not generally added for this step, they can be deducted.
Question 12 (a) (i) $\frac{dV}{dt} = 40\pi \ \text{mm}^3/\text{s}, \ \text{V}=\frac{4}{3}\pi r^3, \ \frac{dV}{dr} = 4\pi r^2$ $\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$ $= \frac{1}{4\pi r^2} \times 40\pi$ $= \frac{10}{9}$ (when $r = 3$)	Done well by most students
(a) (ii) $\text{A}=4\pi r^2, \ \frac{dA}{dr} = 8\pi r$ $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$ $= 8\pi r \times \frac{10}{9}$ $= \frac{80\pi}{3}$ (when $r = 3$)	

Solutions	Marker's Comments
Question 12 (b) (i) 	1 mark for basic shape / concept 1 mark for asymptote $x=\frac{3}{2}$ and the intersection (3,1) and (0,-1). Students needed to take care with scale so that it was clear that the functions intersect when y=1, -1.
(b) (ii) When $x=3,\ y=1$ $1=3a-1$ $a=\frac{2}{3}$	Done well by most students
(c) It is possible that all 12 teams have 3 over age players accounting for $12\times3=36$ players. When the 37th and 38th overage players are considered, then at least one team must have more than 3 players above the age limit. Therefore, by the pigeonhole principle, at least one team will be penalised.	Many students stated that $38\div12=3\cdot1\dot{6}>3$ and concluded that at least one team will have more 3 players and will be penalised. This is a weak response but was awarded the mark this time. Some students gave a stronger justification buy considering 12 teams with 3 over age players=36 but then stated that at **two** teams would be penalised. This is not correct but the subsequent error was ignored this time.
(d) (i) $2\sin x+\sqrt{5}\cos x=R\sin(x+\alpha)$ $R=\sqrt{2^2+\left(\sqrt{5}\right)^2}=3\quad(R>0)$ $\tan\alpha=\frac{\sqrt{5}}{2}$ $\alpha\approx48°11'$ $2\sin x+\sqrt{5}\cos x=3\sin(x+48°11')$	Done well by most students.

(d) (ii) $2\sin x+\sqrt{5}\cos x=1$ $3\sin(x+48°11')=1$ $\sin(x+48°11')=\frac{1}{3}$ $x+48°11'=180°-\sin^{-1}\left(\frac{1}{3}\right),360°+\sin^{-1}\left(\frac{1}{3}\right)$ for $0^0\le\theta\le360^0$ $x=180°-\sin^{-1}\left(\frac{1}{3}\right)-48°11',\ \ 360°+\sin^{-1}\left(\frac{1}{3}\right)-48°11'$ $\approx112°21',\ 331°17'$	Most students got $112°21'$ but many missed $331°17'$
Solutions	**Marker's Comments**
Question 12 (e) (i)  $\overrightarrow{AB}=\begin{pmatrix}-5\\5\end{pmatrix}-\begin{pmatrix}-20\\-12\end{pmatrix}=\begin{pmatrix}15\\17\end{pmatrix}$ $\overrightarrow{OC}=\overrightarrow{OA}+5\times\overrightarrow{AB}$ $=\begin{pmatrix}-20\\-12\end{pmatrix}+5\begin{pmatrix}15\\17\end{pmatrix}$ $=\begin{pmatrix}55\\73\end{pmatrix}$ Hence, the position vector after 5 seconds is $55\underset{\sim}{i}+73\underset{\sim}{j}$.	Lots of errors in this question with multiplying $\overrightarrow{AB}$ by 4 or 5 and matching correctly with A or B. This generated a common incorrect answer of $\begin{pmatrix}10\\22\end{pmatrix}$ which gave an angle of 145^0 in part (ii). Either method is fine so long as it is matched correctly. $\overrightarrow{OC}=\overrightarrow{OB}+4\times\overrightarrow{AB}$ $=\begin{pmatrix}-5\\5\end{pmatrix}+4\begin{pmatrix}15\\17\end{pmatrix}$ $=\begin{pmatrix}55\\73\end{pmatrix}$ Many students incorrectly stated $\begin{pmatrix}75\\85\end{pmatrix}$ which is $\overrightarrow{AC}$ not $\overrightarrow{OC}$ and generates an answer in part (ii) of 162^0.

<table>
<tr>
<td>

(ii)

$$\overrightarrow{OA}\bullet\overrightarrow{OC}=\left|\overrightarrow{OA}\right|\left|\overrightarrow{OC}\right|\cos\theta$$

$$\begin{pmatrix}-20\\-12\end{pmatrix}\bullet\begin{pmatrix}55\\73\end{pmatrix}=\sqrt{(-20)^2+(-12)^2}\sqrt{(55)^2+(73)^2}\times\cos\theta$$

$$\cos\theta=\frac{-20\times55-12\times73}{\sqrt{(-20)^2+(-12)^2}\sqrt{(55)^2+(73)^2}}$$

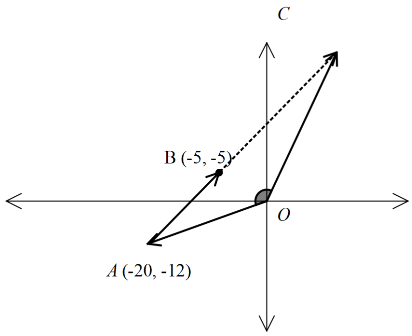
$$\approx-0.926915....$$

$$\theta\approx157°57'32.5"$$

$$\approx158^0 \text{ to the nearest degree}$$

</td>
<td>

Some students use tan ratios in the diagram to find the sum of missing angles. The method works but most students using this method had limited success.



$$\tan\angle COx=\frac{73}{55}\ ,\ \angle COx=53^0$$

$$\angle COy=90^0-53^0=37^0$$

$$\tan\angle AO(-x)=\frac{12}{20},\ \angle AO(-x)=31$$

$$\angle AOC=37^0+90^0+31^0=158^0$$

</td>
</tr>
<tr>
<th>Solutions</th>
<th>Marker's Comments</th>
</tr>
<tr>
<td>

Question 13

(a)

$$\int_{\frac{\pi}{2}}^{\pi}\cos5x\sin3x\ \ dx$$

$$=\frac{1}{2}\int_{\frac{\pi}{2}}^{\pi}(\sin8x-\sin2x)\ \ dx$$

$$=\frac{1}{2}\left[\frac{-\cos8x}{8}+\frac{\cos2x}{2}\right]_{\frac{\pi}{2}}^{\pi}$$

$$=\frac{1}{2}\left(\frac{-\cos8\pi}{8}+\frac{\cos2\pi}{2}-\left(\frac{-\cos4\pi}{8}+\frac{\cos\pi}{2}\right)\right)$$

$$=\frac{1}{2}\left(\frac{-1}{8}+\frac{1}{2}-\left(\frac{-1}{8}+\frac{1}{2}\right)\right)$$

$$=\frac{1}{2}$$

</td>
<td>

Most of the students did well.

</td>
</tr>
</table>

(b) $\dfrac{dy}{dx}=\dfrac{1-x}{y}$ $\int y\,dy=\int(1-x)dx$ $\dfrac{y^2}{2}=x-\dfrac{x^2}{2}+C$ $y^2=2x-x^2+C$ sub $x=2, y=0$ $0=2(2)-2^2+C$ $C=0$ $\therefore y^2=2x-x^2$	Most of the students did well. A few didn't multiply by the y value-they divided instead.
(c) (i) $\overrightarrow{PA}=-7\cos\alpha\boldsymbol{i}+7\sin\alpha\boldsymbol{j}$ $\overrightarrow{PB}=-5\cos(90°-\alpha)\boldsymbol{i}-5\sin(90°-\alpha)\boldsymbol{j}$ $\quad=-5\sin\alpha\boldsymbol{i}-5\cos\alpha\boldsymbol{j}$ $\overrightarrow{PC}=-F\boldsymbol{j}$ $\overrightarrow{PD}=F\boldsymbol{i}$ $F-7\cos\alpha-5\sin\alpha=0$ $F=7\cos\alpha+5\sin\alpha$	Most of the students did well.
Solutions	**Marker's Comments**
(c) (ii) from part i), $-F+7\sin\alpha-5\cos\alpha=0$ $F=7\sin\alpha-5\cos\alpha$ $7\cos\alpha+5\sin\alpha=7\sin\alpha-5\cos\alpha$ $12\cos\alpha=2\sin\alpha$ $\tan\alpha=6$ $\alpha=\tan^{-1}6$ $\alpha=80°32'15.64''$ $\quad\approx 81°$	Most of the students did well.

(d) $y^2 = e^{2x}\sqrt{1+e^{2x}}$ $V = \pi\int_{-1}^{1} e^{2x}\sqrt{1+e^{2x}}\,dx$ $= \frac{\pi}{2}\int_{-1}^{1} 2e^{2x}\left(1+e^{2x}\right)^{\frac{1}{2}}dx$ $= \frac{\pi}{2}\left[\frac{2}{3}\left(1+e^{2x}\right)^{\frac{3}{2}}\right]_{-1}^{1}$ $= \frac{\pi}{3}\left(\left(1+e^{2}\right)^{\frac{3}{2}} - \left(1+e^{-2}\right)^{\frac{3}{2}}\right)$ $= 24.177934.....$ $\approx 24\ \text{unit}^3$	Generally, well done. However, some students didn't square the expression properly and others didn't realise that it was a reverse chain rule with exponentials.
Question 14 (a) $-1 \le \frac{x}{3} \le 1,$ $\therefore$ Domain is $\quad -3 \le x \le 3$ $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ $2\left(-\frac{\pi}{2}\right) + \frac{\pi}{2} \le 2\sin^{-1}\left(\frac{x}{3}\right) + \frac{\pi}{2} \le 2\times\frac{\pi}{2} + \frac{\pi}{2}$ $\therefore$ Range is: $\;-\frac{\pi}{2} \le f(x) \le \frac{3\pi}{2}$	Most of the students did well.

(b) $x=0, y=2\cos^{-1}\left(\frac{1}{2}\right)=\frac{2\pi}{3}$ $y=2\cos^{-1}\left(x+\frac{1}{2}\right)$ $\frac{y}{2}=\cos^{-1}\left(x+\frac{1}{2}\right)$ $\cos\left(\frac{y}{2}\right)=x+\frac{1}{2}$ $x=\cos\left(\frac{y}{2}\right)-\frac{1}{2}$ $A=\int_0^{\frac{2\pi}{3}}\left(\cos\left(\frac{y}{2}\right)-\frac{1}{2}\right)dy$ $=\left[2\sin\left(\frac{y}{2}\right)-\frac{1}{2}y\right]_0^{\frac{2\pi}{3}}$ $=2\sin\left(\frac{1}{2}\times\frac{2\pi}{3}\right)-\frac{1}{2}\times\frac{2\pi}{3}-\left(2\sin\left(\frac{0}{2}\right)-\frac{1}{2}0\right)$ $=2\sin\left(\frac{\pi}{3}\right)-\frac{\pi}{3}$ $=\left(\sqrt{3}-\frac{\pi}{3}\right)$ unit^2	Most of the students did well. Some students didn't convert the function in the subject of *x*. Some didn't find the corresponding *y* value when *x*=0.
(c) $\frac{dV}{dt}=\frac{3(2-V^2)}{V}$ $\int\frac{V}{(2-V^2)}=\int 3dt$ $-\frac{1}{2}\int\frac{-2V}{(2-V^2)}=\int 3dt$ $-\frac{1}{2}\ln\lvert 2-V^2\rvert=3t+C \ (C\in\mathbb{R})$ $\ln\lvert 2-V^2\rvert=-6t+C$ $\lvert 2-V^2\rvert=e^{-6t+C}$ $2-V^2=\pm e^C e^{-6t}$ $2-V^2=Ae^{-6t} \ (A=\pm e^C)$ $V^2=2-Ae^{-6t}$ $V=\sqrt{2-Ae^{-6t}} \ (V>0)$ $t=0, V=0$ $0=2-Ae^{-6(0)}$ $A=2$ $\therefore V=\sqrt{2-2e^{-6x}}$ $t=1,\ \ \mathrm{V}=\sqrt{2-2e^{-6(1)}}=1.412459....\ \approx 1.4\ \mathrm{ms}^{-1}$	Most of the students did well. Some made mistakes finding the integral.

Solutions	Marker's Comments
Question 14 (d) (i) If l and $\underset{\sim}{v}$ overlaps, their gradient will be the same. $l:\ ax+by+c=0$ $y=\frac{-a}{b}x+\frac{c}{b}$ $\therefore\ \underset{\sim}{v}\ =-b\underset{\sim}{i}+a\underset{\sim}{j}$ is one such vector.	Some students did not show sufficient working.
(d) (ii) $\overrightarrow{AO}=\begin{pmatrix}-m\\-n\end{pmatrix}, AO=\left\lvert\overrightarrow{AO}\right\rvert=\sqrt{m^2+n^2}$ The length of $AC=\left\lvert\text{Proj}_{\underset{\sim}{v}}\overrightarrow{AO}\right\rvert=\frac{\overrightarrow{AO}\bullet\underset{\sim}{v}}{\lvert\underset{\sim}{v}\rvert}$ $=\frac{\begin{pmatrix}-m\\-n\end{pmatrix}\bullet\begin{pmatrix}-b\\a\end{pmatrix}}{\sqrt{(-b)^2+a^2}}$ $=\frac{mb-na}{\sqrt{a^2+b^2}}$ In $\triangle OAC, OC^2=AO^2-AC^2$ $=\left(\sqrt{m^2+n^2}\right)^2-\left(\frac{mb-na}{\sqrt{a^2+b^2}}\right)^2$ $=\frac{(m^2+n^2)(a^2+b^2)-m^2b^2+2mnab-n^2a^2}{a^2+b^2}$ $=\frac{m^2a^2+m^2b^2+n^2a^2+n^2b^2-m^2b^2+2mnab-n^2a^2}{a^2+b^2}$ $=\frac{m^2a^2+2mnab+n^2b^2}{a^2+b^2}$ $=\frac{(ma+nb)^2}{a^2+b^2}$ (1) since $A(m,n)$ is on the line l, substitute $x=m,\ y=n$ into $ax+by+c=0$ $am+bn+c=0$ $am+bn=-c$ (2) substitute (2) into (1): $OC^2=\frac{(-c)^2}{a^2+b^2}$ $\therefore OC=\frac{\lvert c\rvert}{\sqrt{a^2+b^2}}$	Some students managed to get the length of AC correctly using projection. Only a few students show the full working to get OC. Instead of using Pythagoras, you can find the vector OC and then find the length. You also need to justify why $am+bn=c$.